Compressed Radar via Doppler Focusing

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Abstract— **We investigate the problem of a monostatic pulse-Doppler radar transceiver trying to detect targets, sparsely populated in the radar's unambiguous time-frequency region. Several past works employ compressed sensing (CS) algorithms to this type of problem, but either do not address sample rate reduction, impose constraints on the radar transmitter, propose CS recovery methods with prohibitive dictionary size, or perform poorly in noisy conditions. Here we describe a sub-Nyquist sampling and recovery approach called Doppler focusing which performs low rate sampling and digital processing, imposes no restrictions on the transmitter, and uses a CS dictionary with size which does not increase with number of pulses P. Furthermore, in the presence of noise, Doppler focusing enjoys a signal-to-noise ratio (SNR) improvement which scales linearly with P, obtaining good detection performance even at SNR as low as -25dB. It can easily incorporate clutter rejection capabilities, and handle targets with large dynamic range. The recovery is based on the Xampling framework, which allows sub-Nyquist analog-to-digital conversion. The entire digital recovery process is also performed at the low rate. Finally, our approach is implemented in hardware using a Xampling radar prototype.**

I. Introduction

We consider target detection and parameter estimation in a pulse-Doppler radar system, using sub-Nyquist sampling. The radar is a single transceiver, monostatic, narrow-band system. Targets are non-fluctuating point targets, sparsely populated in the radar's unambiguous time-frequency region. We propose a recovery method which can detect and estimate targets' time delay and Doppler frequency, using a linear, non-adaptive sampling technique at a rate significantly lower than the radar signal's Nyquist frequency.

Current state-of-the-art radar systems sample at the signal's Nyquist rate, which can be hundreds of MHz and even up to several GHz. Systems exploiting sub-Nyquist sampling benefit from a lower rate analog-to-digital conversion (ADC), which requires less power consumption, heat dissipation, and cost. Our goal is to present some steps in order to break the link between radar signal bandwidth and sampling rate. The sub-Nyquist Xampling [1], [2] method we use for this purpose is an ADC which performs analog prefiltering of the signal before taking point-wise samples. These compressed samples ("Xamples") contain the information needed to recover the signal parameters using compressed sensing (CS) algorithms.

Past works employ CS algorithms to this type of problem, but do not address sample rate reduction and continue sampling at the Nyquist rate [3]. Other works combine radar and CS in order to reduce the receiver's sampling rate, but in doing so impose constraints on the radar transmitter and do not treat noise [4], or do not handle noise well [5]. The work in [5] first estimates target delays and then uses these recovered delays to estimate Doppler frequencies and amplitudes. Another line of work proposes single stage CS recovery methods with dictionary size proportional to the product of delay and Doppler grid sizes, making them infeasible for many realistic scenarios [3], [6].

At the crux of our proposed recovery method is a coherent superposition of time shifted and modulated pulses, the Doppler focusing function $\Phi(t;\nu)$. For any Doppler frequency ν, this function combines the received signals from different pulses so targets with appropriate Doppler frequencies come together in phase. For each sought after ν, $\Phi(t;\nu)$ is processed as a simple one-dimensional CS problem and the appropriate time delays are recovered. The gain from this method is both in terms of SNR and Doppler resolution, and it can be carried out on the low rate samples, as will be shown in Section III.

Using our approach we acquire the sub-Nyquist samples and then digitally recover the unknown target parameters using low rate processing, without returning to the higher Nyquist rate. Our Doppler focusing based method separates the Doppler from delay recovery, as opposed to many CS delay-Doppler estimation methods which depend upon constructing a CS dictionary with a column for each delay-Doppler hypothesis, and suffering from a dictionary explosion problem. The SNR achieved using Doppler focusing scales linearly with the number of received pulses P, as does an optimal MF, providing good performance in SNR as low as -25dB, as will be shown in Section IV. Regarding clutter, Doppler focusing includes inherent isolation between targets with different Doppler frequencies, so unless target and clutter have very similar Doppler frequency, target detection is unhindered.

II. Radar Model

We consider a radar transceiver that transmits a pulse train

$$x_T(t) = \sum_{p=0}^{P-1} h(t - p\tau), \quad 0 \le t \le P\tau \tag{1}$$

consisting of P equally spaced pulses $h(t)$. The pulse-to-pulse delay τ is referred to as the PRI, and its reciprocal $1/\tau$ is the PRF. The entire span of the signal in (1) is called the coherent processing interval (CPI). The pulse $h(t)$ is a known time-limited baseband function with continuous-time Fourier transform (CTFT) $H(\omega) = \int_{-\infty}^{\infty} h(t)e^{-j\omega t}dt$. We assume that $H(\omega)$ has negligible energy at frequencies beyond $B_h/2$ and we refer to B_h as the bandwidth of $h(t)$. The target scene is composed of L non-fluctuating point targets (Swerling-0

978-1-4799-2035-8/14/$31.00@2014 IEEE

model, see [7]), where we assume that L is known, although this assumption can easily be relaxed. The pulses reflect off the L targets and propagate back to the transceiver. Each target ℓ is defined by three parameters: a time delay τ_ℓ, proportional to the target's distance from the radar; a Doppler radial frequency ν_ℓ, proportional to the target-radar closing velocity; and a complex amplitude α_ℓ, proportional to the target's radar cross section (RCS) and all other propagation factors.

Assuming the signal is narrowband, we can write the received signal as

$$x(t) = \sum_{p=0}^{P-1} \sum_{\ell=0}^{L-1} \alpha_\ell h(t - \tau_\ell - p\tau) e^{-j\nu_\ell p\tau}. \quad (2)$$

It will be convenient to express the signal as a sum of single frames $x(t) = \sum_{p=0}^{P-1} x_p(t)$, where

$$x_p(t) = \sum_{\ell=0}^{L-1} \alpha_\ell h(t - \tau_\ell - p\tau) e^{-j\nu_\ell p\tau}. \quad (3)$$

Our goal in this work is to accurately detect the L targets, *i.e.* to estimate the $3L$ DOF $\{\alpha_\ell, \tau_\ell, \nu_\ell\}_{\ell=0}^{L-1}$ in (2), using the least possible number of digital samples.

III. Delay-Doppler Recovery

We begin by describing how Xampling can be performed on the multi pulse signal (2). We then describe Doppler focusing based recovery using these Xamples, and analyze two aspects of the algorithm: the effect of multiple pulses on SNR when noise exists, and the minimal number of samples required for perfect recovery without noise. Finally we discuss some practical considerations and clutter.

A. Xampling

Since $x_p(t)$ is confined to the interval $t \in [p\tau, (p+1)\tau]$, it can be expressed by its Fourier series

$$x_p(t) = \sum_{k \in \mathbb{Z}} c_p[k] e^{j2\pi kt/\tau}, \quad t \in [p\tau, (p+1)\tau], \quad (4)$$

where

$$\begin{aligned} c_p[k] &= \frac{1}{\tau} \int_{p\tau}^{(p+1)\tau} x_p(t) e^{-j2\pi kt/\tau} dt \\ &= \frac{1}{\tau} \sum_{\ell=0}^{L-1} \alpha_\ell e^{-j\nu_\ell p\tau} \int_{p\tau}^{(p+1)\tau} h(t - \tau_\ell - p\tau) e^{-j2\pi kt/\tau} dt \\ &= \frac{1}{\tau} H(2\pi k/\tau) \sum_{\ell=0}^{L-1} \alpha_\ell e^{-j\nu_\ell p\tau} e^{-j2\pi k\tau_\ell/\tau}. \quad (5) \end{aligned}$$

Past works [2], [8], [9] have shown how these Fourier coefficients can be obtained from the time domain signal $x(t)$. From (5) we see that all $3L$ unknown parameters $\{\alpha_\ell, \tau_\ell, \nu_\ell\}_{\ell=0}^{L-1}$ are embodied in the Fourier coefficients $c_p[k]$ in the form of a complex sinusoid problem. The number of Fourier coefficients sampled in each pulse, $|\kappa|$, controls the trade-off between sample rate and robustness to noise.

B. Applying Doppler Focusing and CS Recovery

Having acquired $c_p[k]$ using Xampling, we now create a weighted sum of Fourier coefficients, *i.e.* perform the Doppler focusing operation for a specific frequency ν:

$$\begin{aligned} \Psi_\nu[k] &= \sum_{p=0}^{P-1} c_p[k] e^{j\nu p\tau} \\ &= \frac{1}{\tau} H(2\pi k/\tau) \sum_{\ell=0}^{L-1} \alpha_\ell e^{-j2\pi k\tau_\ell/\tau} \sum_{p=0}^{P-1} e^{j(\nu-\nu_\ell)p\tau}. \quad (6) \end{aligned}$$

We now analyze the sum of exponents in (6). For any given ν, targets with Doppler frequency ν_ℓ in a band of width $2\pi/P\tau$ around ν, *i.e.* in $\Psi_\nu[k]'s$ "focus zone", will achieve coherent integration and an SNR boost of approximately

$$g(\nu|\nu_\ell) = \sum_{p=0}^{P-1} e^{j(\nu-\nu_\ell)p\tau} \overset{|\nu-\nu_\ell|<\pi/P\tau}{\cong} P \quad (7)$$

compared with a single pulse. On the other hand, since the sum of P equally spaced points covering the unit circle is generally close to zero, targets with ν_ℓ not "in focus" will approximately cancel out. Thus $g(\nu|\nu_\ell) \cong 0$ for $|\nu - \nu_\ell| > \pi/P\tau$. Therefore, Doppler focusing performed on the low rate sub-Nyquist samples obtains:

$$\Psi_\nu[k] \cong \frac{P}{\tau} H(2\pi k/\tau) \sum_{\{\ell:|\nu-\nu_\ell|<\pi/P\tau\}} \alpha_\ell e^{-j2\pi k\tau_\ell/\tau}. \quad (8)$$

To analyze the effect of Doppler focusing on SNR, we add noise to (2):

$$\tilde{x}(t) = x(t) + w(t), \quad (9)$$

where $w(t)$ is a zero mean wide-sense stationary random signal with autocorrelation $r_w(s) = \sigma^2\delta(s)$. The Fourier coefficients in (4) then become $\tilde{c}_p[k] = c_p[k] + w_p[k]$, where $w_p[k]$ is a zero mean complex random variable with variance σ^2/τ. It can be shown [10] that the SNR in the coefficients after focusing is P times greater than before Doppler focusing. Thus, we have obtained a linear SNR improvement with P, as does an optimal MF.

For each ν we now have a delay estimation problem, which can be written as

$$\mathbf{\Psi}_\nu = \frac{P}{\tau} \mathbf{H}\mathbf{V}\mathbf{x}_\nu \quad (10)$$

where $\mathbf{H}$ is a $|\kappa| \times |\kappa|$ diagonal matrix with elements $H(2\pi k_i/\tau)$ and $\mathbf{V}$ is a $|\kappa| \times N_\tau$ Vandermonde matrix with $\mathbf{V}_{mq} = e^{-j2\pi k_m n/N_\tau}$. Assuming target delays are integer multiples of τ/N_τ, $\mathbf{x}_\nu$ is L-sparse and

$$\mathbf{\Psi}_\nu = [\Psi_\nu[k_0] \; ... \; \Psi_\nu[k_{|\kappa|-1}]]^T \in \mathbb{C}^{|\kappa|}. \quad (11)$$

Note that (10) forms a CS problem, which can be solved with standard CS methods.

The Doppler focusing operation (6) is a continuous operation on the variable ν, and can be performed for any Doppler frequency up to the PRF. With Doppler focusing there are no inherent "blind speeds", *i.e.* target velocities which are undetectable, as occurs with classic Moving Target Indication (MTI) [11]. Define the set of Fourier coefficients $C = \{c_p[k]\}_{0 \le p < P}^{k \in \kappa}$, and $\mathbf{\Psi}_\nu(C)$ as the vector of focused coefficients (11) obtained from C using (6). Therefore $\mathbf{x}_\nu(C)$, can be recovered from $\mathbf{\Psi}_\nu(C)$ for any ν. Since strong amplitudes are indicative of true target existence as opposed to noise, Doppler focusing recovery searches for large values of $|\mathbf{x}_\nu(C)[n]|$ and estimates target delays and Doppler frequencies as $n\Delta_\tau$ and ν accordingly. After detecting each target, its influence is removed from the set of Fourier coefficients in order to reduce masking of weaker targets and to remove spurious targets created by processing sidelobes. A similar subtraction is performed in many iterative algorithms such as Orthogonal Matching Pursuit. Detection is performed iteratively until all targets have been detected, if L is known, or until an amplitude threshold is met, if the model order is unknown.

C. Noiseless Recovery

The following theorems analyze the minimal number of samples required for perfect recovery when there is no noise. Proofs are given in [10].

Theorem 1 *The minimal number of samples required for perfect recovery of L targets when there is no noise, is at least $4L^2$, with $|\kappa|$ and P at least $2L$ each.*

This result coincides with the minimal sampling rate for two dimensional spectral analysis [12].

Theorem 2 *Suppose target Doppler frequencies are aligned to a grid $\{\tilde{\nu}_m = 2\pi m/\tau M\}_{m=-M/2}^{M/2-1}$, with no restriction on target delays. Then the minimal number of samples required for perfect recovery of L targets when there is no noise, is $2L\min(M, 2L)$.*

Theorem 3 *Under the conditions of Theorem 2, the minimal number of samples required for perfect recovery of L targets using Doppler focusing is $2LM$, with $|\kappa| \ge 2L$ and $P \ge M$.*

The minimal rate requirement exists separately on the number of sampled Fourier coefficients $|\kappa|$ and the number of sampled pulses P, and not for their product. This shows that in terms of minimal sampling rate, samples in the coefficient dimension k cannot be replaced by samples in the pulse dimension p, and vice versa.

These theorems show that the requirement of Doppler focusing for $|\kappa| \ge 2L$ matches the general lower bound on the number of samples required in each pulse. Furthermore, when $M = O(L)$, the number of pulses required for Doppler focusing is within order of magnitude of the lower bound.

D. Practical Considerations and Clutter

If one wishes to probe a uniform grid of M Doppler frequencies, *i.e.* $\{\tilde{\nu}_m = 2\pi m/\tau M\}_{m=-M/2}^{M/2-1}$, then $\Psi_\nu[k]$ can be created efficiently using a length M DFT:

$$\Psi_{\tilde{\nu}_m}[k] = \sum_{p=0}^{P-1} c_p[k] e^{j2\pi mp/M} = DFT_M\{c_p[k]\}.$$

Another practical concern is target dynamic range. Since target amplitudes can differ by several orders of magnitude, care must be taken so strong targets do not mask weaker ones. When focusing on some Doppler frequency ν, targets with Doppler frequencies ν_ℓ satisfying $|\nu_\ell - \nu| > \pi/P\tau$ are undesirable. These targets can be viewed as "out-of-focus". We can add to (6) a user defined window function $w[p]$ (*e.g.* Hann, Blackman, etc.) which is designed to mitigate the impact of these out-of-focus targets:

$$\begin{aligned}\Psi_\nu[k] &= \sum_{p=0}^{P-1} c_p[k] e^{j\nu p\tau} w[p] \\ &= \frac{1}{\tau} H(2\pi k/\tau) \sum_{\ell=0}^{L-1} \alpha_\ell e^{-j2\pi k\tau_\ell/\tau} \sum_{p=0}^{P-1} e^{j(\nu-\nu_\ell)p\tau} w[p]. \quad (12)\end{aligned}$$

In Fig. 1 we see an example of how windowing can reduce the effect of out-of-focus targets compared with no windowing (constant $w[p]$).

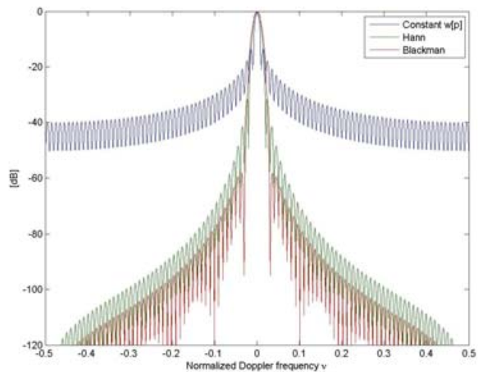

Fig. 1: DFT of windowing functions $w[p]$ compared with no windowing (constant $w[p]$) for $P = 100$ pulses.

Finally, we show that Doppler focusing possesses inherent clutter rejection capabilities, suggesting that special prefiltering operations such as MTI may not be required. The most common method to allow detection in clutter ridden scenarios, is to utilize the fact that clutter, as opposed to most targets, is mostly static. If we assume the radar transceiver itself is also stationary, then clutter echoes will be received with zero Doppler frequency. This is the reason that classic anti-clutter methods (*e.g.* MTI) are basically a notch filter blocking the Doppler frequency generated by the radar's own motion. Doppler focusing includes inherent target-clutter Doppler separation, so it does not require any prefiltering in order to allow target detection when facing clutter.

The Doppler focusing operation (6) can be viewed as passing the Xamples $c_p[k]$ through a bandpass filter bank, where each filter has a pass-band of width $2\pi/P\tau$. The filters' attenuation can be controlled using windowing (12), at the cost of increasing the pass-band width. This creates adjustable isolation between delay estimation problems (10) for targets with Doppler frequencies separated by more than the pass-band width. Therefore, if clutter were to be primarily concentrated around some specific frequency, targets with Doppler frequencies shifted away by more than approximately $2\pi/P\tau$ could be detected without interference.

IV. Simulation Results and Radar Experiment

We now present some numerical experiments illustrating the recovery performance of a sparse target scene. We corrupt the received signal $x(t)$ with an additive white Gaussian noise $n(t)$ with power spectral density $S_n(f) = N_0/2$, bandlimited to $x(t)$'s bandwidth B_h. We define the signal to noise power ratio for target ℓ as $\mathrm{SNR}_\ell = \frac{1}{T_p}\int_0^{T_p} |\alpha_\ell h(t)|^2 dt / N_0 B_h$, where T_p is the pulse time. The scenario parameters used were number of targets L=5, number of pulses P=100, PRI τ=10μsec, and B_h=200MHz. Target delays and Doppler frequencies are spread uniformly at random in the appropriate unambiguous regions, and target amplitudes were chosen with constant absolute value and random phase. The classic time and frequency resolutions ("Nyquist bins"), defined as $1/B_h$ and $1/P\tau$, are $5n$sec and 1 KHz accordingly.

In order to demonstrate a 1:10 sampling rate reduction, our sub-Nyquist Xampling scheme generated 200 Fourier coefficients per pulse, as opposed to the 2000 Nyquist rate samples. We compared Doppler focusing with classic processing and a two-stage recovery method as described in [5] using a "hit-rate" criterion: we define a "hit" as a delay-Doppler estimate which is circumscribed by an ellipse around the true target position in the time-frequency plane.

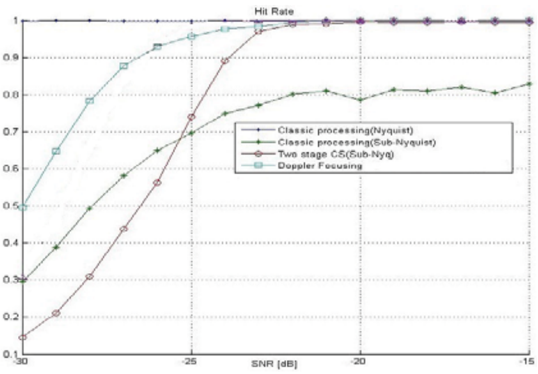

Fig. 2: Hit Rate for classic processing, two-stage CS recovery and Doppler focusing. Sub-Nyquist sampling rate was one tenth the Nyquist rate.

Fig. 2 demonstrates the hit-rate performance of the different recovery methods. It is evident that Doppler focusing is superior to the other sub-Nyquist recovery techniques. Other simulations in [10] show that the delay and Doppler estimations errors using Doppler focusing are very close to the Nyquist rate errors, that Doppler focusing is able to distinguish between closely spaced targets, and that Doppler focusing's performance improves when the transmitted waveform's energy is concentrated in the sampled frequencies.

We now present a real experiment of our radar receiver hardware prototype. Our setup includes a custom made sub-Nyquist radar receiver board (see Fig. 3) which implements sub-Nyquist Xampling and digital recovery using Doppler focusing, while AWR software is used to simulate the reflections from several targets. The analog input signal (2) was synthesised using National Instruments (NI) hardware. This signal is sampled as described in [2], and target parameters are successfully estimated digitally using Doppler focusing.

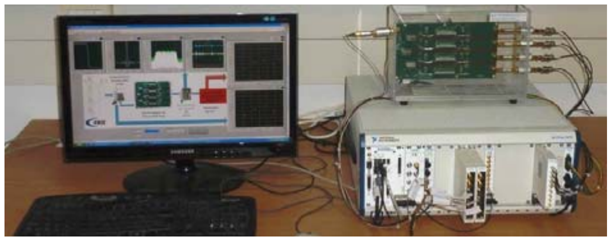

Fig. 3: The 4-channel radar receiver board on top of the NI chassis, and a PC with the LabView demo screen.

To evaluate the board we make use of NI PXI equipment for both system synchronization and signal sources. The entire component ensemble, wrapped in the NI chassis, is depicted in Fig. 3. The RF front end and board we use are identical to the ones used in [2], but the digital recovery method accounting for target Doppler frequencies is different.

This experimental prototype proves that the sub-Nyquist methodology described in this paper is actually feasible in practice. The recovery method proposed here not only describes digital recovery, but also addresses the problem of sampling the analog signal at a low rate, in a way which is feasible with standard RF hardware.

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